CONSOLIDATION OF A POROUS ELASTIC SOIL STRATUM SUBJECTED TO AXISYMMETRIC SURFACE LOADING

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ABSTRACT

In this study, the consolidation of a porous elastic soil stratum in welded contact with elastic medium subjected to axisymmetric surface loads has been investigated. The compressible fluid and solid constituents are considered. Permeability is different in horizontal as well as vertical directions for porous medium. The Laplace-Hankel transform technique is utilized for finding the analytical expressions for the displacements caused by normal circular loading. The extended Simpson's and Schapery's formula are utilized for the Hankel and Laplace inversion, respectively. For numerical computations, the continental, as well as the oceanic layer of the Earth, is considered. The effect of the vertical displacement with time for both layers is discussed. It is observed that the compressibility of the solid components increases the surface settlement for both layers. It is also observed that for different values of anisotropic permeability, the initial and final settlement is the same.

KEYWORDS: Consolidation, Poroelastic, Soil Stratum, Axisymmetric, Surface Loads

INTRODUCTION

For saturated soil, there is a decrease in value under compressive force, which is known as the compressibility of soil. Compression is the process that illustrates the decrease in soil volume under an externally applied load. The soil particles squeeze, when stress is applied to a water saturated soil, the water will flow out of the soil. If water comes out from pores then air can't take place. This compression of a saturated soil due to a steady static pressure is termed as consolidation. The process of soil consolidation was first discovered by Terzaghi (1). He suggested a fundamental approach to the study of a fully saturated soil and expanded the one-dimensional theory of soil consolidation. Rendulic (2) generalized this theory to three-dimensional case. Booker and Small (3) developed a method for the analysis of the consolidation of a horizontally layered soil using Fourier transform method. Booker and Small (4) extended the results of Booker and Small (3) for solving the problem of consolidation of horizontally layered soils under both axially symmetric and general surface loading.

Selvadurai and Yue (5) examined the axisymmetric consolidation response of a poroelastic layer lying on a rigid impermeable base caused by circular foundation. Mei et al. (6) considered a cross-anisotropic elastic constitutive model for the consolidation of a layered soil using the finite layer procedure. Chen et al. (7) presented an exact analytical solution for the axisymmetric consolidation of a transversely isotropic, semi-infinite saturated soil. Conte (8) presented an effective technique for the study of coupled consolidation in unsaturated soils underneath plane strain with axially symmetric conditions. Singh et al. (9) discussed the quasi-static deformation of a half-space subjected to axisymmetric surface loads. Menendez (10) discussed the consolidation of an elastic saturated soil with incompressible fluid using the finite element method. Rani et al.

(11) studied the influence of negative Poisson's ratio on the deformation of a half-space for normal strip loading as well as axisymmetric normal disc loading.

Kebli and Merzouk (12) provided an analytical expression of an axisymmetric deformation of an elastic layer overlying on a rigid support containing a circular hole due to to uniform loading. Ai and Cheng (13) built up the precise integration method for the investigation of consolidation behaviors of multilayered isotropic media. Cheng and Ai (14) provided the solution for three-dimensional consolidation of transversely isotropic layered saturated soils in the cylindrical coordinate system. Lu et al. (15) developed the reflection-transmission matrix method to study the axisymmetric consolidation of a layered transversely isotropic saturated soil. Ai and Wang (16) obtained the solutions for multilayered poroelastic soils caused by an axisymmetric effect of Biot's compressibility constraints.

Ai et al. (17) studied the transient interpretation of a transversely isotropic multilayered half-space subjected to vertical loading. Li and Cui (18) presented the solutions for the consolidation problem of saturated soils by pumping. Ai et al. (19) investigates the multi-dimensional consolidation of transversely isotropic viscoelastic saturated soils. Closed-form solutions for one-dimensional consolidation in saturated soils under external loading has been discussed by Deng et al. (20). The consolidation for isotropic viscoelastic media with compressible component because of tangential circular loads has been investigated by Ai et al. (21). Wang et al. (22) presented plane strain semi-analytical solutions for consolidation of unsaturated soil under time-dependent loading. Lo et al. (23) presented theory for a double-layer system consisting upper unsaturated and lower saturated zones caused by surface load.

The aim of this study is to investigate the consolidation of a homogeneous, isotropic, poroelastic soil stratum in welded contact with an elastic medium under axisymmetric surface loading. The fluid and the solid constituents are chosen to be compressible. The case of normal circular loading is considered. Laplace-Hankel transform technique is utilized for finding the analytical expressions for the displacements. The influence of the vertical displacement with time in the poroelastic soil stratum has been discussed. The findings may be employed in Geotechnical and Geophysical Engineering.

GOVERNING EQUATIONS FOR POROELASTIC SOIL STRATUM

For axisymmetric consolidation, the governing equations are (24)

$$\frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r}\frac{\partial u_r}{\partial r} - \frac{u_r}{r^2} + \frac{\partial^2 u_r}{\partial z^2} + \frac{1}{1-2\nu}\frac{\partial e}{\partial r} - \frac{\alpha}{G}\frac{\partial \sigma}{\partial r} = 0 \tag{1}$$

$$\frac{\partial^2 u_z}{\partial r^2} + \frac{1}{r}\frac{\partial u_z}{\partial r} + \frac{\partial^2 u_z}{\partial z^2} + \frac{1}{1-2\nu}\frac{\partial e}{\partial z} - \frac{\alpha}{G}\frac{\partial \sigma}{\partial z} = 0 \tag{2}$$

where

$$e = \frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z}, \text{ denotes dilatation.} \tag{3}$$

u_r, u_z are the displacements in r, z direction, respectively. σ denotes the pore-pressure. G denotes the shear modulus, α is the Biot-Willis coefficient, ν is the drained Poisson's ratio.

The fluid flux $q = (q_r, q_\theta, q_z)$ is given by

$$q_r = -\chi_r \, \partial\sigma/\partial r \, , \; q_\theta = 0, \; q_z = -\chi_z \, \partial\sigma/\partial z \quad (4)$$

where $q = (q_r, q_\theta, q_z)$ and (χ_r, χ_z) be the Darcy conductivity in (r, z) direction.

The constitutive equations are

$$\sigma_{rr} = 2G\left[e_{rr} + \frac{v}{(1-2v)}e\right] - \alpha\sigma$$

$$\sigma_{\theta\theta} = 2G\left[e_{\theta\theta} + \frac{v}{(1-2v)}e\right] - \alpha\sigma$$

$$\sigma_{zz} = 2G\left[e_{zz} + \frac{v}{(1-2v)}e\right] - \alpha\sigma$$

$$\sigma_{rz} = 2Ge_{rz}, \; \sigma_{r\theta} = \sigma_{\theta z} = 0$$

$$\zeta = \alpha e + \frac{1}{M}\sigma \quad (5)$$

where

$$M = \frac{2G(v_u - v)}{\alpha^2(1-2v)(1-2v_u)} \quad \text{, is the Biot modulus.} \quad (6)$$

$\sigma_{rr}, \sigma_{\theta\theta}, \sigma_{zz}$ are the normal stresses in r, θ, z direction, respectively and $\sigma_{rz}, \sigma_{r\theta}, \sigma_{\theta z}$ are the corresponding shear stresses. ζ is the increment of fluid content and v_u is undrained Poisson's ratio.

The equation of compatibility is obtained from equations (1)-(3)

$$\eta\nabla^2\sigma = G\nabla^2 e \quad\quad\quad (7)$$

where

$$\eta = \frac{(1-2v)}{2(1-v)}\alpha \quad , \quad \nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} \quad \text{is the Laplacian operator.}$$

The continuity equation is defined as

$$\frac{\partial\zeta}{\partial t} = -div\, q \quad\quad\quad (8)$$

The combination of three equations i.e., the continuity equation, the Darcy's law and constitutive equations gives the fluid diffusion equation which is written in the form

$$\chi_r\left(\frac{\partial^2\sigma}{\partial r^2}+\frac{1}{r}\frac{\partial\sigma}{\partial r}\right)+\chi_z\frac{\partial^2\sigma}{\partial z^2}=\frac{\partial}{\partial t}\left(\alpha e+\frac{1}{M}\sigma\right) \tag{9}$$

SOLUTION OF THE GOVERNING EQUATIONS

The n$^{\text{th}}$ order Hankel transform $\overline{f}\left(k,z,s\right)$ of $\tilde{f}\left(r,z,s\right)$ is defined as

$$\overline{f}\left(k,z,s\right)=\int_0^\infty \tilde{f}\left(r,z,s\right)J_n\left(kr\right)r\,dr \tag{10}$$

so that

$$\tilde{f}\left(r,z,s\right)=\int_0^\infty \overline{f}\left(k,z,s\right)J_n\left(kr\right)k\,dk \tag{11}$$

where $J_n\left(kr\right)$ is the nth order Bessel function.

The Laplace transform is defined as

$$\tilde{f}\left(r,z,s\right)=\int_0^\infty f\left(r,z,t\right)e^{-st}dt, \; s \text{ is the Laplace transform parameter.} \tag{12}$$

Application of the Laplace transform to equation (1) gives

$$\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}-\frac{1}{r^2}\right)\tilde{u}_r+\frac{\partial^2\tilde{u}_r}{\partial z^2}+\frac{1}{1-2\nu}\frac{\partial\tilde{e}}{\partial r}-\frac{\alpha}{G}\frac{\partial\tilde{\sigma}}{\partial r}=0 \tag{13}$$

where $\tilde{u}_r$, $\tilde{e}$, $\tilde{\sigma}$ are the variables in Laplace transform domain corresponding to u_r, e, σ, respectively.

Taking 1$^{\text{st}}$ order Hankel transform of equation (13) which yields

$$\frac{d^2\overline{u}_r}{dz^2}-k^2\overline{u}_r-\frac{1}{1-2\nu}k\,\overline{e}+\frac{\alpha}{G}k\,\overline{\sigma}=0 \tag{14}$$

where $\overline{u}_r$, $\overline{e}$, $\overline{\sigma}$ are the variables in Hankel transform domain corresponding to $\tilde{u}_r$, $\tilde{e}$, $\tilde{\sigma}$ respectively.

Equations (7) and (9) implies

$$\left[c_r(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r})+c_z\frac{\partial^2}{\partial z^2}-\frac{\partial}{\partial t}\right]\nabla^2\sigma=0 \tag{15}$$

which gives

$$\overline{\sigma}=B_1e^{-mz}+C_1e^{mz}+B_2e^{-kz}+C_2e^{kz} \tag{16}$$

where B_1, C_1, B_2, C_2 are arbitrary constants.

Equations (7) and (16) gives

$$\bar{e} = \frac{\eta}{G}\left(B_1 e^{-mz} + C_1 e^{mz}\right) + B_3 e^{-kz} + C_3 e^{kz} \tag{17}$$

where B_3, C_3 are arbitrary constants.

Substituting equations (16) and (17) into equation (14) which gives

$$\bar{u}_r = -\frac{\eta k}{G\left(m^2 - k^2\right)}\left(B_1 e^{-mz} + C_1 e^{mz}\right) + z\left(B_6 e^{-kz} - C_6 e^{kz}\right) + B_4 e^{-kz} + C_4 e^{kz} \tag{18}$$

Similarly, taking the Laplace transform followed by zeroth order Hankel transform of equation (2) and using equations (16) and (17) which implies

$$\bar{u}_z = -\frac{\eta m}{G\left(m^2 - k^2\right)}\left(B_1 e^{-mz} - C_1 e^{mz}\right) + z\left(B_6 e^{-kz} + C_6 e^{kz}\right) + B_5 e^{-kz} - C_5 e^{kz} \tag{19}$$

Hence, the complete solution of governing equations for poroelastic soil stratum in Laplace transform domain can be written as:

$$\tilde{\sigma}(r,z,s) = \int_0^\infty \left(B_1 e^{-mz} + C_1 e^{mz} + B_2 e^{-kz} + C_2 e^{kz}\right) J_0(kr) k \, dk \tag{20}$$

$$\tilde{e}(r,z,s) = \int_0^\infty \left[\frac{\eta}{G}\left(B_1 e^{-mz} + C_1 e^{mz}\right) + B_3 e^{-kz} + C_3 e^{kz}\right] J_0(kr) k \, dk \tag{21}$$

$$\tilde{q}_r(r,z,s) = \chi_r \int_0^\infty \left(B_1 e^{-mz} + C_1 e^{mz} + B_2 e^{-kz} + C_2 e^{kz}\right) J_1(kr) k^2 \, dk \tag{22}$$

$$\tilde{q}_z(r,z,s) = \chi_z \int_0^\infty \left(m B_1 e^{-mz} - m C_1 e^{mz} + k B_2 e^{-kz} - k C_2 e^{kz}\right) J_0(kr) k \, dk \tag{23}$$

$$\tilde{u}_r(r,z,s) = \int_0^\infty \left[-\frac{\eta k}{G\left(m^2 - k^2\right)}\left(B_1 e^{-mz} + C_1 e^{mz}\right) + z\left(B_6 e^{-kz} - C_6 e^{kz}\right)\right.$$

$$\left. + B_4 e^{-kz} + C_4 e^{kz}\right] J_1(kr) k \, dk \tag{24}$$

$$\tilde{u}_z(r,z,s) = \int\limits_0^\infty \left[-\frac{\eta m}{G(m^2-k^2)}\left(B_1 e^{-mz} - C_1 e^{mz}\right) + z\left(B_6 e^{-kz} + C_6 e^{kz}\right) \right.$$

$$\left. +B_5 e^{-kz} - C_5 e^{kz} \quad \right] J_0(kr)k \; dk \tag{25}$$

$$\tilde{\sigma}_{rz}(r,z,s) = 2G\int\limits_0^\infty \left[\frac{\eta mk}{G(m^2-k^2)}\left(B_1 e^{-mz} - C_1 e^{mz}\right) - kz\left(B_6 e^{-kz} + C_6 e^{kz}\right) \right.$$

$$\left. +\left(\frac{1}{2}B_3 - kB_4\right)e^{-kz} - \left(\frac{1}{2}C_3 - kC_4\right)e^{kz} \quad \right] J_1(kr)k \; dk \tag{26}$$

$$\tilde{\sigma}_{zz}(r,z,s) = 2G\int\limits_0^\infty \left[\frac{\eta k^2}{G(m^2-k^2)}\left(B_1 e^{-mz} + C_1 e^{mz}\right) - B_6(1+kz)e^{-kz} - C_6(1-kz)e^{kz} \right.$$

$$\left. +\left(\frac{1}{2}B_3 - kB_4\right)e^{-kz} + \left(\frac{1}{2}C_3 - kC_4\right)e^{kz} \quad \right] J_0(kr)k \; dk \tag{27}$$

where the unknowns $B_j, C_j \; (j=1,2,4)$ are calculated by the application of the boundary conditions.

$$\begin{pmatrix} B_3 \\ C_3 \end{pmatrix} = \frac{\eta}{G}\left[1-(1-2v)\frac{(1-v_u)s_a}{(v_u-v)s}\right]\begin{pmatrix} B_2 \\ C_2 \end{pmatrix}$$

$$\begin{pmatrix} B_5 \\ C_5 \end{pmatrix} = \begin{pmatrix} B_4 \\ C_4 \end{pmatrix} + \frac{\eta}{2Gk}\left[-1+(3-4v)\frac{(1-v_u)s_a}{(v_u-v)s}\right]\begin{pmatrix} B_2 \\ C_2 \end{pmatrix}$$

$$\begin{pmatrix} B_6 \\ C_6 \end{pmatrix} = \frac{\eta}{2G}\left[1+\frac{(1-v_u)s_a}{(v_u-v)s}\right]\begin{pmatrix} B_2 \\ C_2 \end{pmatrix}$$

$$s_a = s + (c_r - c_z)k^2 = c_z(m^2 - k^2) \tag{28}$$

SOLUTION OF THE GOVERNING EQUATIONS FOR THE ELASTIC MEDIUM

The governing equations for the elastic medium are (25)

$$\frac{\partial^2 u_{rH}}{\partial r^2} + \frac{\partial}{\partial r}\left(\frac{u_{rH}}{r}\right) + \frac{\partial^2 u_{rH}}{\partial z^2} + \frac{1}{1-2v_H}\frac{\partial e_H}{\partial r} = 0 \tag{29}$$

$$\frac{\partial^2 u_{zH}}{\partial r^2} + \frac{1}{r}\frac{\partial u_{zH}}{\partial r} + \frac{\partial^2 u_{zH}}{\partial z^2} + \frac{1}{1-2v_H}\frac{\partial e_H}{\partial z} = 0 \tag{30}$$

where

$$e_H = div\,\mathbf{u} \quad = \frac{\partial u_{rH}}{\partial r} + \frac{u_{rH}}{r} + \frac{\partial u_{zH}}{\partial z} \tag{31}$$

u_{rH}, u_{zH} are displacements in r, z direction, respectively. v_H be the drained Poisson's ratio.

The constitutive equations are

$$\sigma_{rrH} = 2G_H\left[e_{rrH} + \frac{v_H}{(1-2v_H)}e_H\right] \quad \sigma_{zzH} = 2G_H\left[e_{zzH} + \frac{v_H}{(1-2v_H)}e_H\right]$$

$$\sigma_{rzH} = 2G_H e_{rzH} \tag{32}$$

where

$\sigma_{rrH}, \sigma_{zzH}$ are the normal stresses in r, z direction, respectively and σ_{rzH} is the corresponding shear stress. G_H denotes the shear modulus.

Using equations (29) and (30) and taking zeroth order Hankel transform, a suitable solution is given as

$$\bar{e}_H = D_1 e^{-kz} \tag{33}$$

where $\bar{e}_H$ is the variables in Hankel transform domain corresponding to e_H.

Taking 1st order Hankel transform of equation (29) which yields

$$\bar{u}_{rH} = \left[-\frac{z}{2(1-2v_H)}D_1 + D_2\right]e^{-kz} \tag{34}$$

Similarly, zeroth order Hankel transform of equation (30) gives

$$\bar{u}_{zH} = \left[-\frac{z}{2(1-2v_H)}D_1 + D_3\right]e^{-kz} \tag{35}$$

and

$$kD_3 = \frac{1}{2(1-2v_H)}\{4v_H - 3\}D_1 + kD_2 \tag{36}$$

where $\bar{u}_{rH}, \bar{u}_{zH}$ are the variables in Hankel transform domain corresponding to u_{rH}, u_{zH} respectively. D_1, D_2, D_3 are arbitrary constants.

Hence, the complete solution of the governing equations for the elastic medium is

$$u_{rH}(r,z) = \int_0^\infty \left[-\frac{z}{2(1-2\nu_H)} D_1 + D_2 \right] e^{-kz} J_1(kr) k \, dk \tag{37}$$

$$u_{zH}(r,z) = \int_0^\infty \left[\frac{1}{2k(1-2\nu_H)} \{(4\nu_H - 3) + kz\} D_1 + D_2 \right] e^{-kz} J_0(kr) k \, dk \tag{38}$$

$$\sigma_{rzH}(r,z) = 2G_H \int_0^\infty \left[\frac{1}{2(1-2\nu_H)} \{(1-2\nu_H) + kz\} D_1 - kD_2 \right] e^{-kz} J_1(kr) k \, dk \tag{39}$$

$$\sigma_{zzH}(r,z) = 2G_H \int_0^\infty \left[\frac{1}{2(1-2\nu_H)} \{2(1-\nu_H) + kz\} D_1 - kD_2 \right] e^{-kz} J_0(kr) k \, dk \tag{40}$$

BOUNDARY CONDITIONS

Consider a homogeneous, isotropic, porous elastic soil stratum of thickness d lying over a homogeneous, isotropic, elastic medium as shown in Figure 1. Assume the upper surface of the porous elastic soil stratum is permeable and the normal stress $(\sigma_{zz})_0$ is applied on it which implies

$$\sigma_{zz} = (\sigma_{zz})_0, \quad \sigma_{rz} = 0, \quad \sigma = 0 \qquad \text{at} \quad z = -d \tag{41}$$

It is also assumed that along the plane $z = 0$, the porous elastic soil stratum lying over an elastic medium which gives

$$\sigma_{rz} = \sigma_{rzH}; \ \sigma_{zz} = \sigma_{zzH}; \ u_r = u_{rH}; \ u_z = u_{zH} \quad \text{at} \quad z = 0 \tag{42}$$

Also, the interface ($z = 0$) is assumed to be impermeable, we have

$$q_z = 0 \qquad\qquad \text{at} \quad z = 0 \tag{43}$$

Let

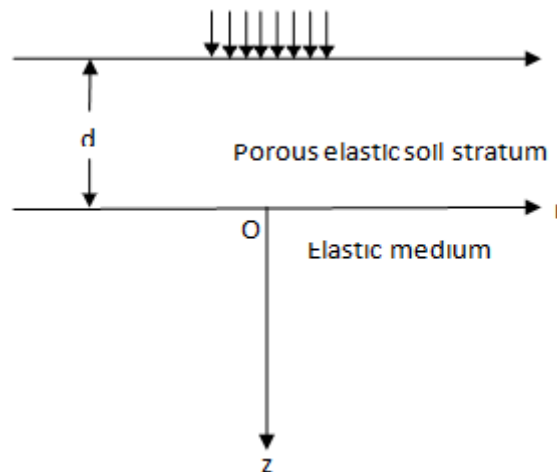$$(\sigma_{zz})_0 = \int_0^\infty N_0 J_0(kr) k \, dk. \tag{44}$$

Figure 1: Geometry of the porous Elastic Soil Stratum Lying Over an Elastic Medium.

The equations (20), (23)-(27), (37)-(40) together with equations (41)-(43) generate a system of eight equations. The constants $B_j, C_j (j=1,2,4)$, $D_j (j=1,2)$ can be calculated by solving these equations. After substituting the values of $B_j, C_j (j=1,2,4)$ in equations (20), (23)-(27), (37)-(40), the expressions are obtained in the Laplace-Hankel transforms domain. The explicit expression for the vertical displacement at $(r=z=0)$ is given as

$$\tilde{u}_z(0,0,s) = -\frac{Q_0}{2G'\pi l}\int_0^\infty b_1\Big[\big\{-2kb_5(b_7+b_8)e^{2kd}+2kb_5(b_7-b_8)e^{-2kd}$$

$$-2\big[b_1+(b_2+b_9)b_7+(b_1+b_9+b_{11})b_8\big]e^{-(m-k)d}$$

$$+2\big[b_1+(b_2-b_9)b_7-(b_1-b_9+b_{12})b_8\big]e^{-(m+k)d}$$

$$-2\big[b_1+(b_2+b_{10})b_7+(b_1+b_{10}+b_{12})b_8\big]e^{(m+k)d}$$

$$+2\big[b_1+(b_2-b_{10})b_7-(b_1-b_{10}+b_{11})b_8\big]e^{(m-k)d}$$

$$+12kb_5b_8\big]\frac{J_1(lk)}{kD}\,dk \qquad\qquad (45)$$

where

$$G' = G_H/G$$

$$D = 8kb_5(b_7+b_8)\big\{-b_1+(b_7-b_8)(b_1-b_3+kb_5)\big\}e^{kd}$$

$$+8kb_5(b_7-b_8)\big\{-b_1+(b_7+b_8)(b_1+b_3+kb_5)\big\}e^{-kd}$$

$$+\left\{-4b_1\left(b_1-b_7\left(b_4+kb_5\right)-\frac{k^2}{m}b_5b_8\right)+\left(b_7^2-b_8^2\right)\right.$$

$$\left.\left\{-2b_4\left(b_4+kb_5\right)-4b_{10}\left(b_3-\frac{k^2}{m}b_5\right)-2\left(kb_5\left(b_1+b_2\right)+2b_1b_2\right)\right\}\right\}e^{md}$$

$$+\left\{-4b_1\left(b_1-b_7\left(b_4+kb_5\right)+\frac{k^2}{m}b_5b_8\right)-\left(b_7^2-b_8^2\right)\right.$$

$$\left.\left\{2b_4\left(b_4+kb_5\right)+4b_9\left(b_3+\frac{k^2}{m}b_5\right)+2\left(kb_5\left(b_1+b_2\right)+2b_1b_2\right)\right\}\right\}e^{-md}$$

$$+\left(b_7+b_8\right)\left(b_1+b_2-\left(m-k\right)b_5\right)\left\{2b_1-\left(b_7-b_8\right)\left(b_4+b_{12}\right)\right\}e^{(m+2k)d}$$

$$+\left(b_7+b_8\right)\left(b_1+b_2+\left(m+k\right)b_5\right)\left\{2b_1-\left(b_7-b_8\right)\left(b_4+b_{11}\right)\right\}e^{-(m-2k)d}$$

$$+\left(b_7-b_8\right)\left(b_1+b_2+\left(m+k\right)b_5\right)\left\{2b_1-\left(b_7+b_8\right)\left(b_4+b_{11}\right)\right\}e^{(m-2k)d}$$

$$+\left(b_7-b_8\right)\left(b_1+b_2-\left(m-k\right)b_5\right)\left\{2b_1-\left(b_7+b_8\right)\left(b_4+b_{12}\right)\right\}e^{-(m+2k)d} \quad (46)$$

and

$$b_1=\frac{\left\{2(1-\nu)(1-\nu_u)\right\}s_a}{(\nu_u-\nu)s}, \quad b_2=1-\frac{\{1-2\nu\}b_1}{2(1-\nu)},$$

$$b_3=\left\{1+\frac{1}{2(1-\nu)}b_1\right\}kd, \quad b_4=-1+\frac{(3-4\nu)b_1}{2(1-\nu)},$$

$$b_5=\frac{2k}{m^2-k^2}, \quad b_6=\frac{2kG}{\eta},$$

$$b_7=1-\frac{\{1-2\nu_H\}G'}{(3-4\nu_H)}, \quad b_8=\frac{\{2(1-\nu_H)\}G'}{(3-4\nu_H)},$$

$$b_9=b_3+mb_5, \; b_{10}=b_3-mb_5, \; b_{11}=\frac{k}{m}(m+k)b_5, \; b_{12}=\frac{k}{m}(m-k)b_5 \quad (47)$$

The integral expression for the displacements is similar to the expression for a poroelastic half-space presented by Singh et al. (9) for the case $d\to\infty$.

Numerical Analysis

In this study, the solution is obtained in the Laplace-Hankel transform domain. The extended Simpson's formula has been utilized for the Hankel inversion. For the evaluation of the Laplace inversion, Schapery's formula has been used. It is an easy and more efficient method for computation proposed by Schapery (26) which has been utilized in the problems related to consolidation of soil. It is defined as

$$\Psi(t) = \left[s\tilde{\Psi}(s) \right]_{s=(1/2t)} \tag{48}$$

where $\tilde{\Psi}(s)$ is the Laplace transform of $\Psi(t)$ and $\left[\ \right]_{s=(1/2t)}$ indicates that the value of s is (1/2t).

For computation of the vertical displacement at the centre of the disc load, the following dimensionless quantities are assumed

$$W = \left(\pi l G/Q_0 \right) u_z\left(0,0\right),$$

$$T = \left(G\chi_z/l^2 \right)t, \qquad \gamma^2 = c_r/c_z = \chi_r/\chi_z \tag{49}$$

Also, the consolidation rate is defined as

$$\text{Consolidation rate} = \frac{u_z(t) - u_z(0)}{u_z(\infty) - u_z(0)} \tag{50}$$

For numerical computations, the continental, as well as the oceanic layer, is considered. For the continental layer, the poroelastic soil stratum is chosen to be of Westerly Granite for which $v = 0.25, v_u = 0.34, B = 0.85$ (27) and for oceanic layer, the poroelastic soil stratum chosen to be of Hanford Basalt for which $v = 0.30, v_u = 0.31, B = 0.12$ (27). The rigidity ratios for the continental and oceanic layers are $G_H/G = 2.2$ and $G_H/G = 1.76$, respectively (28). Assuming the elastic medium is Poissonian i.e. $v_H = 0.25.$

Figures 2 and 6 depict the influence of compressibility of solid components on vertical displacement at $r = z = 0$ for both continental as well as oceanic layer. $\alpha = 1,$ denotes the incompressibility of the solid components. It is observed that the compressibility of the solid components increases the surface settlement for both layers.
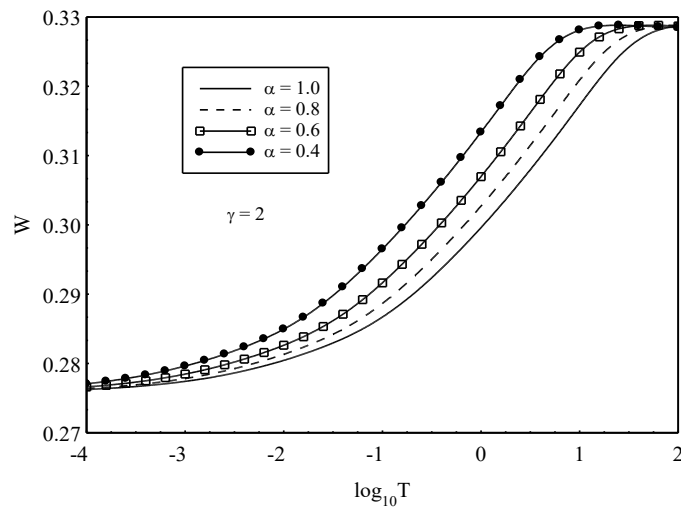
**Figure 2: Influence of the Compressibility of Solid Constituents on the
Time-Settlement Behavior for Continental Layer.**

The effect of undrained Poisson's ratio $\nu_u \left(-1 \leq \nu \leq \nu_u \leq 0.5 \right)$ on the vertical displacement for the continental

as well oceanic layer is shown in Figures 3 and 7 for different values of ν_u. $\nu_u = 0.5$, denotes the incompressibility of

fluid constituent. These figures reveal that for the undrained state $\left(T \rightarrow 0 \right)$, incompressibility of fluid constituent

decreases the vertical displacement but for drained state the vertical displacement has no effect.
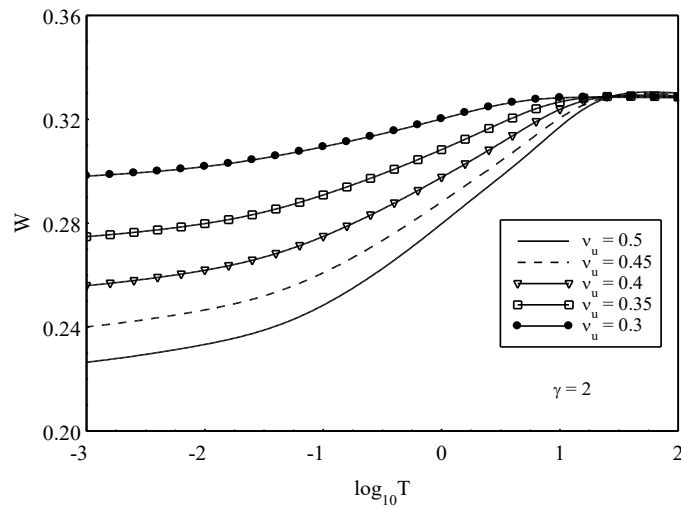
**Figure 3: Influence of the Compressibility of Fluid Constituents on Vertical
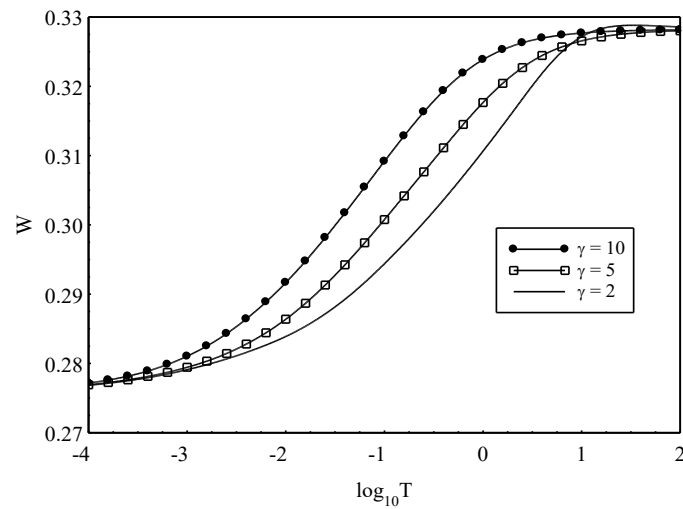Displacement at the Origin for Continental Layer.**

Figure 4: Time-Settlement Behavior for Different Values of Anisotropic Permeability Parameter γ for Continental Layer.

Time-settlement behavior for different values of anisotropic permeability parameter γ for the continental as well as oceanic layer is represented in Figures 4 and 8. Large value of anisotropic permeability (γ) makes faster the surface settlement for both the layers. It is also observed that for different values of anisotropic permeability, the initial and final settlement is same.
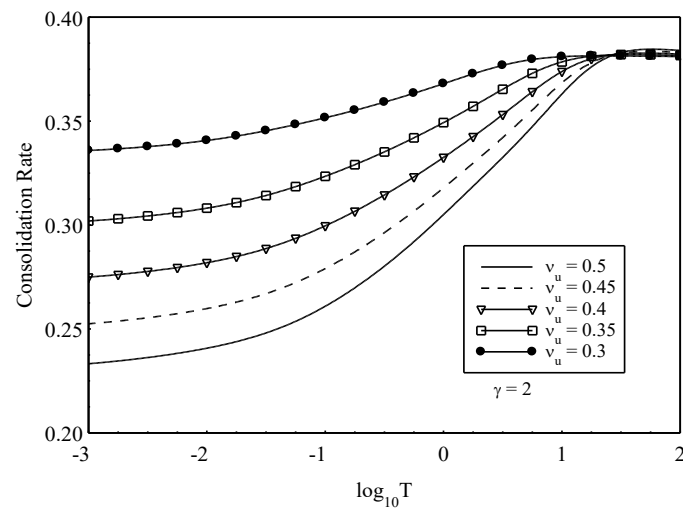

Figure 5. Effect of the Compressibility of Fluid Constituents on Consolidation Rate for Continental Layer.
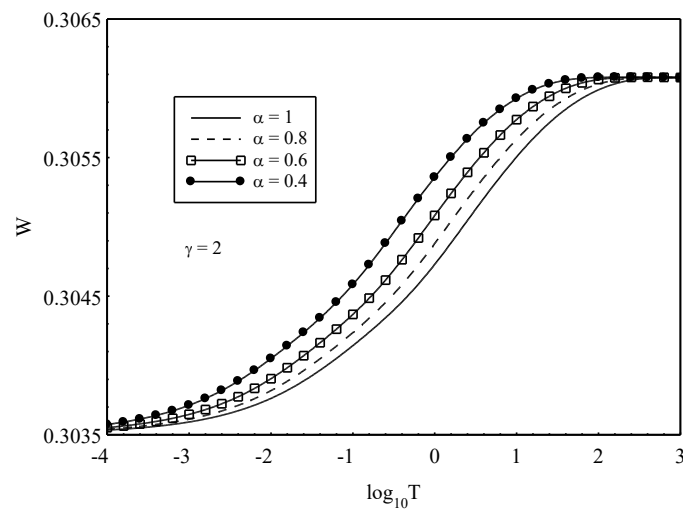
**Figure 6. Influence of the Compressibility of Solid Constituents
on the Time-Settlement Behavior for Oceanic Layer.**
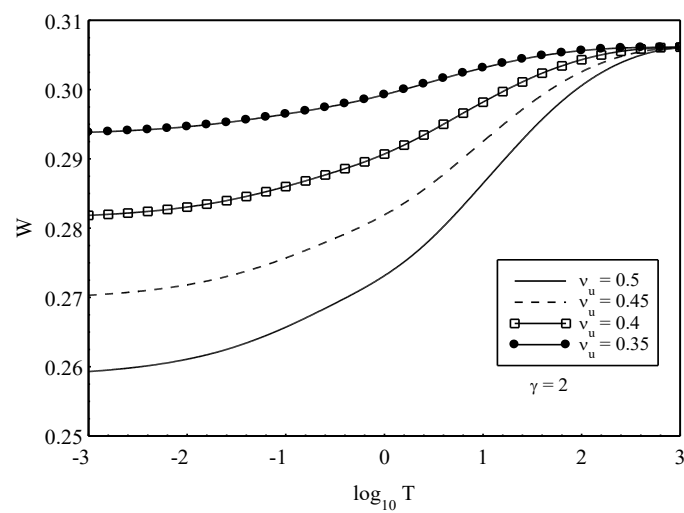

**Figure 7. Influence of the Compressibility of Fluid
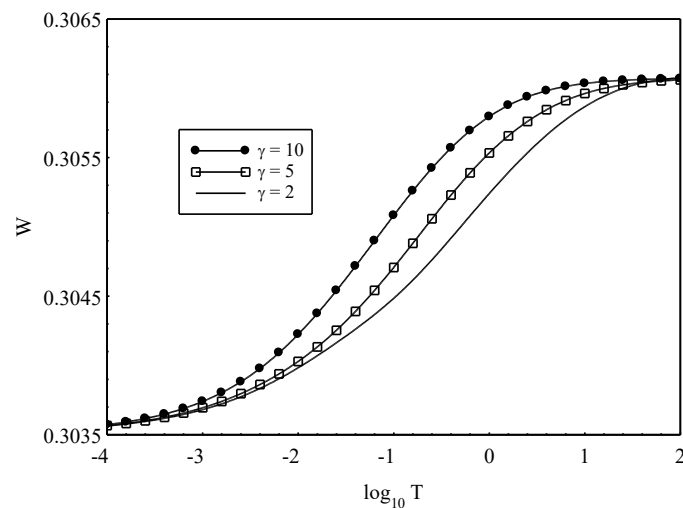Constituents on Vertical Displacement at the Origin for
Oceanic Layer.**

**Figure 8. Time-Settlement Behavior for Different Values of
Anisotropic Permeability Parameter γ for Oceanic Layer.**

The influence of the compressibility of fluid constituents on the consolidation rate for continental as well as oceanic layer is represented in Figures 5 and 9. It shows that the compressibility of fluid constituents have significant effect on consolidation rate for both layers. In case of undrained state, the consolidation rate for the incompressible fluid constituent is less than the compressible fluid constituent.
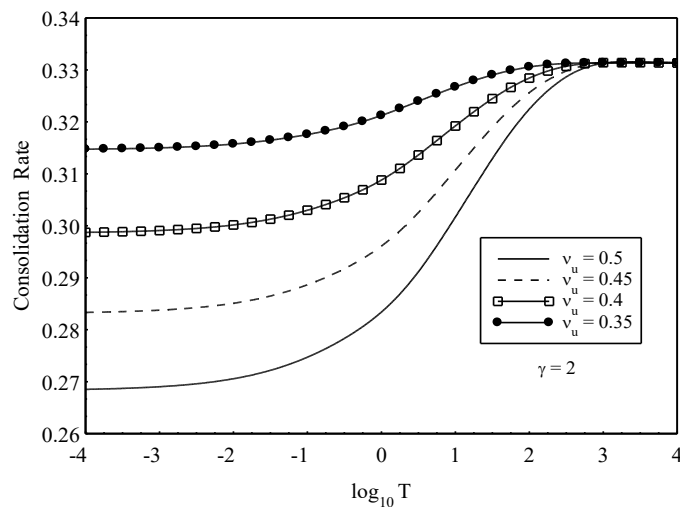

**Figure 9. Effect of the compressibility of fluid constituents
on consolidation rate for oceanic layer.**

CONCLUSIONS

- The compressibility of the solid components increases the surface settlement.

- Incompressibility of fluid constituent decreases the vertical displacement.

- Large value of anisotropic permeability $\left(\gamma\right)$ makes faster the surface settlement.

- For different values of anisotropic permeability, the initial and final settlement is same.

- Compressibility of fluid components has a significant effect on consolidation rate.

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